Topology

B. Math. II

Supplementary Examination

Instructions: All questions carry equal marks.

1. Define open maps and closed maps between topological spaces. Show by examples that an open map need be a closed map and a closed map need not be an open map. Also given an example of a map that is both open and closed and an example of a map that is neither open nor closed.

2. Define the product topology and the uniform topology on $\mathbb{R}^\omega$, the countable product of the real line $\mathbb{R}$. Consider the subset $S \subset \mathbb{R}^\omega$ consisting of all sequences that has only finitely many non-zero terms. Compute its closure in both of these topologies.

3. Define connected topological space. Prove that the product of connected spaces is connected.

4. Let $f : X \to Y$ be a map between two topological spaces. Assume Y is compact Hausdorff. Then prove that f is continuous if and only if its graph,
$$G_f = \{(x, f(x)) \mid x \in X$$
is a closed sunset of $X \times Y$.

5. Define second countable space. Prove that any uncountable subset of a second countable space must have uncounatbly many limit points.

6. Define normal topological space. Prove that every compact Hausdorff space is normal. Is the converse true? Justify your answer.